ECE317 : Feedback and Control

Lecture:
Block Diagrams

Dr. Richard Tymerski
Dept. of Electrical and Computer Engineering
Portland State University

Transfer function (review)

- A transfer function is defined by

$$G(s) := \frac{Y(s)}{R(s)}$$

Laplace transform of system output

Laplace transform of system input

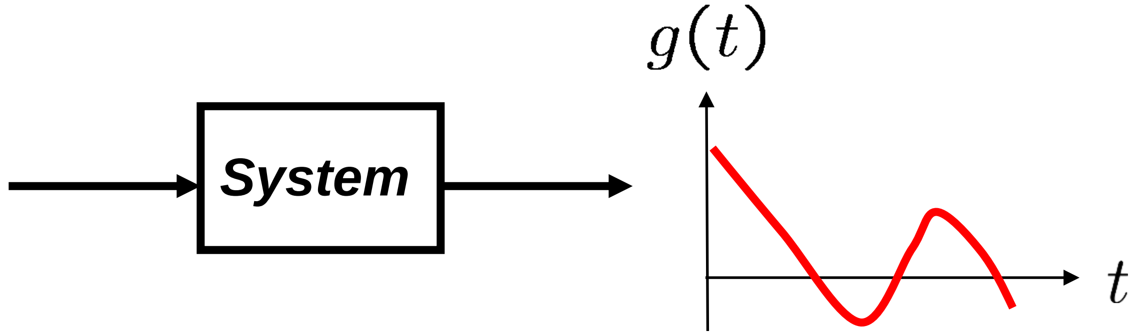
$$R(s) \xrightarrow[\text{input}]{} \boxed{G(s)} \xrightarrow[\text{output}]{} Y(s) = G(s)R(s)$$

- A system is assumed to be at rest. (zero initial condition)
- Transfer function is a generalization of “gain” concept.

Impulse response (review)

- Suppose that *r(t)* is the unit impulse function and system is at rest.

$$r(t) = \delta(t)$$
$$R(s) = 1$$



- The output *g(t)* for the unit impulse input is called *unit impulse response*.
- Since *R(s)=1*, the transfer function can also be defined as the Laplace transform of impulse response:

$$G(s) := \mathcal{L}\{g(t)\}$$

Course roadmap

Modeling

- Laplace transform
- Transfer function
- Block Diagram Linearization
- Models for systems
  - electrical
  - mechanical
  - example system

Analysis

Stability
- Pole locations
- Routh-Hurwitz

Time response
- Transient
- Steady state (error)

Frequency response
- Bode plot

Design

Design specs

Frequency domain

Bode plot

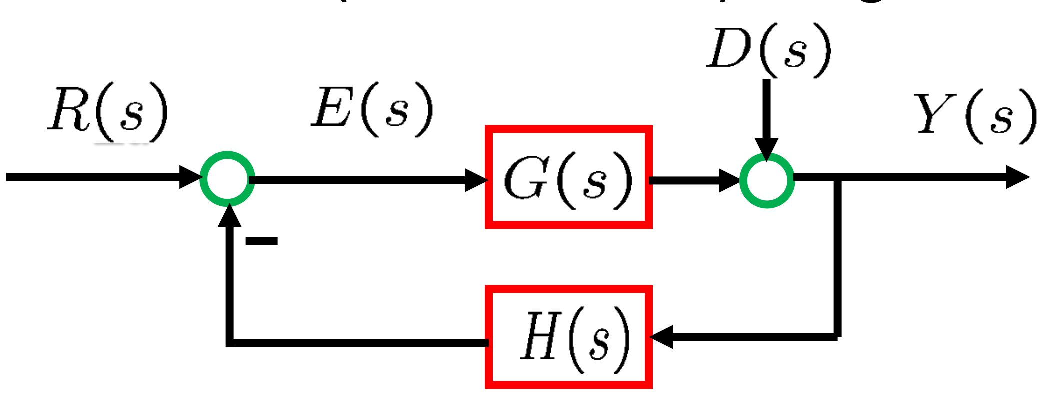
Compensation

Design examples

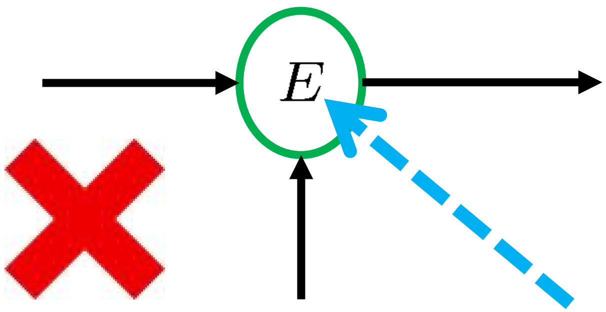
Matlab & PECS simulations & laboratories

Block diagram

- Represents relations among signals and systems
- Very useful in control systems
- Also useful in computer simulations (Simulink)
- Elements
  - Block: transfer function ("gain" block)
  - Arrow: signal
  - Node: summation (or subtraction) of signals

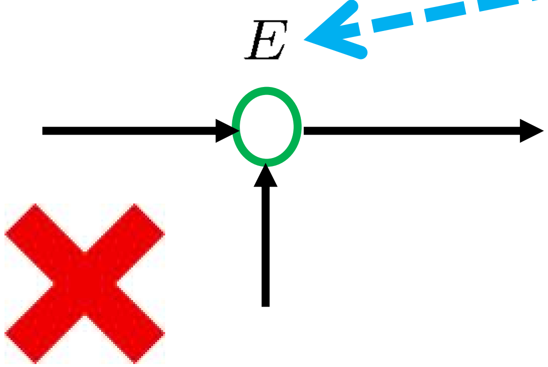


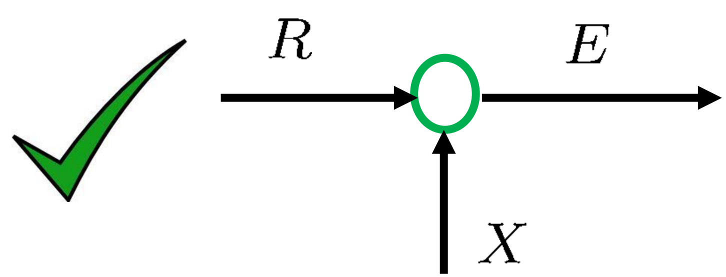
Typical mistakes



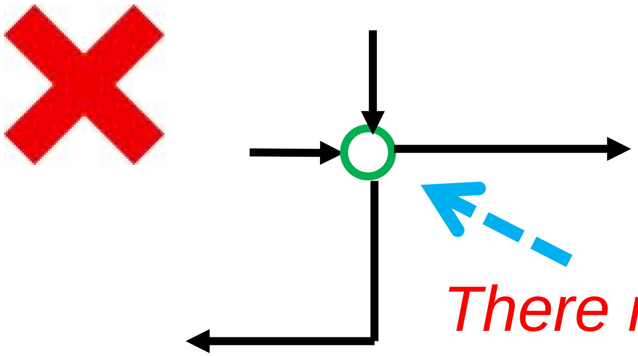
Unclear which signal is "E"

Signal must be indicated on an arrow.



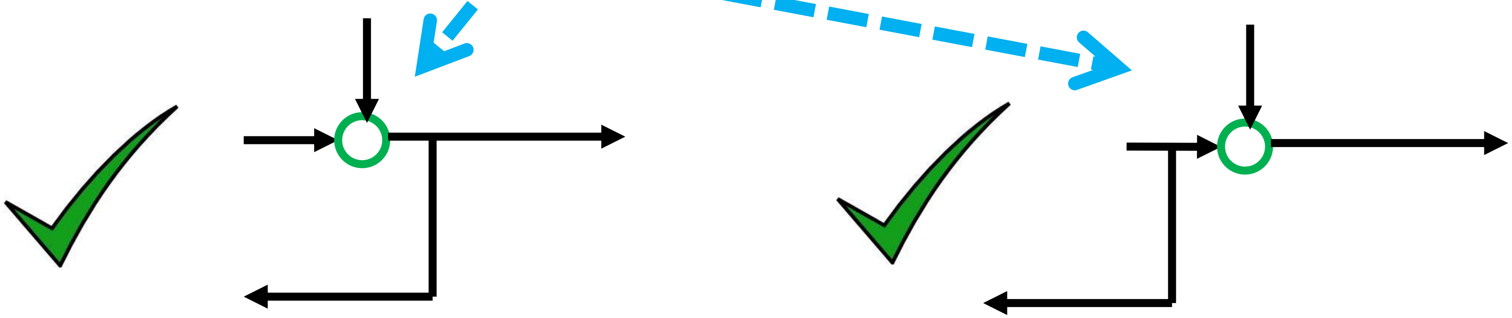


Typical mistakes (cont'd)



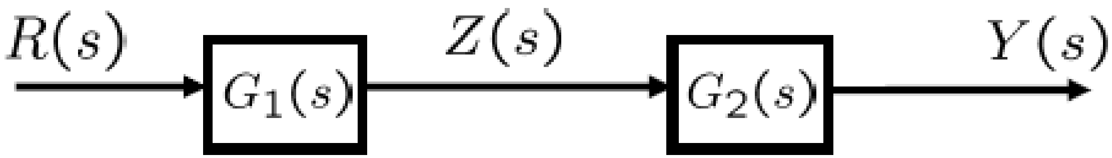
There must be only one output from a node.

Both are fine, but they have different meanings!



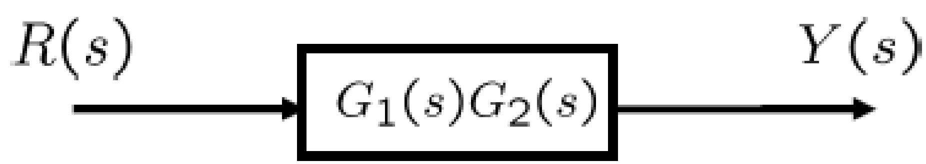
Elementary TF block diagrams

- Series connection



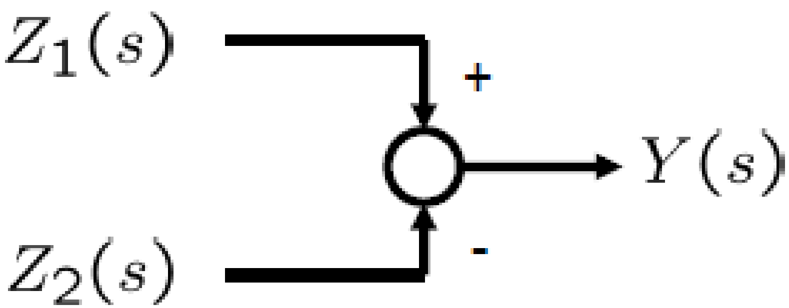
$$\frac{Z(s)}{R(s)} = G_1(s) \qquad \frac{Y(s)}{Z(s)} = G_2(s)$$

$$\Rightarrow \quad \frac{Y(s)}{R(s)} = G_1(s)G_2(s)$$



Elementary TF block diagrams

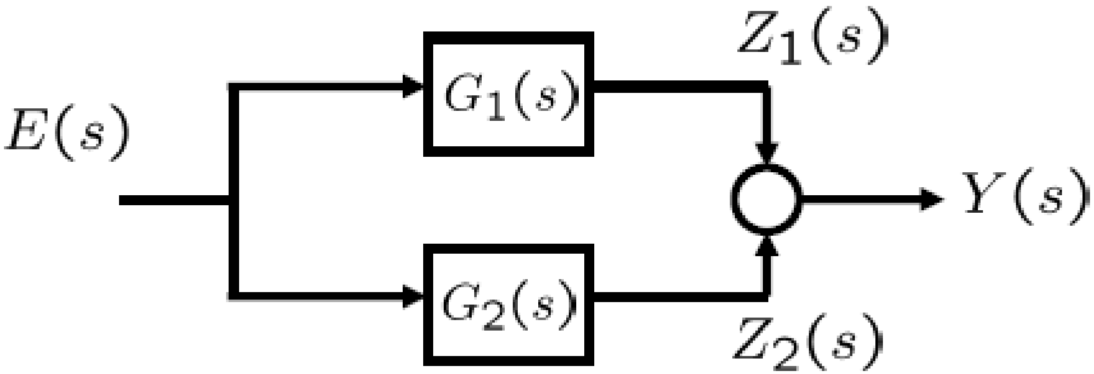
- Summing Junction



$$\Rightarrow \quad Y(s) = Z_1(s) - Z_2(s)$$

Elementary TF block diagrams

- Parallel connection

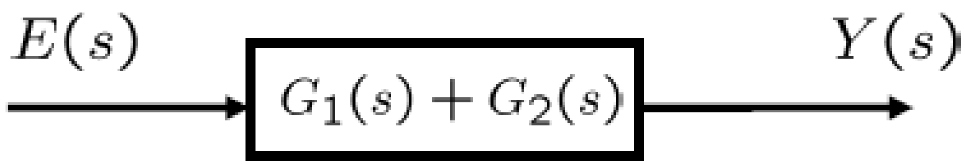


$$\frac{Z_1(s)}{E(s)} = G_1(s)$$

$$\frac{Z_2(s)}{E(s)} = G_2(s)$$

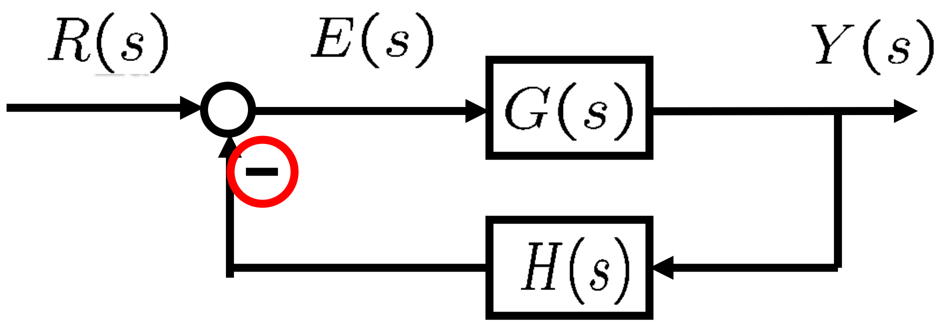
$$Y(s) = Z_1(s) + Z_2(s) = (G_1(s) + G_2(s))E(s)$$

$$\frac{Y(s)}{E(s)} = G_1(s) + G_2(s)$$



Transfer function (TF) with feedback

- Negative feedback system

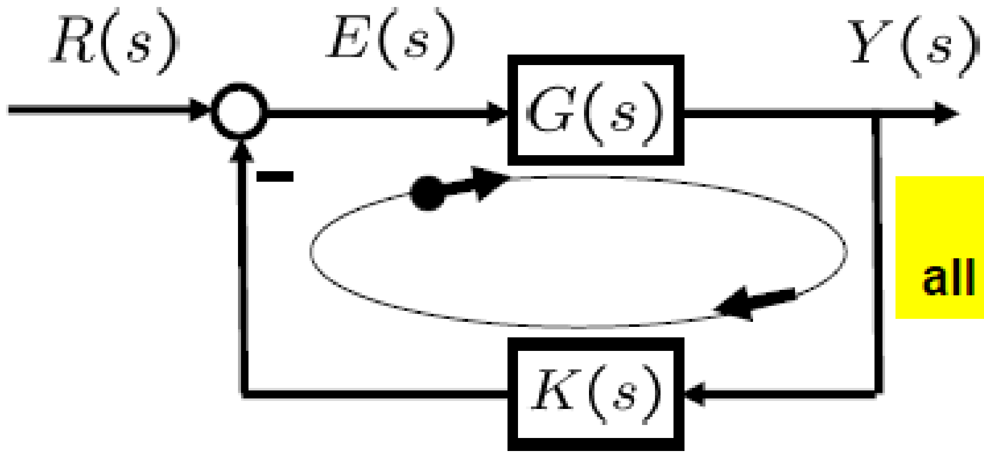


$$E(s) = R(s) - H(s)G(s)E(s) \quad \Rightarrow \quad E(s) = \frac{1}{1+G(s)H(s)}R(s)$$

$$Y(s) = G(s)E(s) \quad \Rightarrow \quad Y(s) = \boxed{\frac{G(s)}{1+G(s)H(s)}}R(s)$$

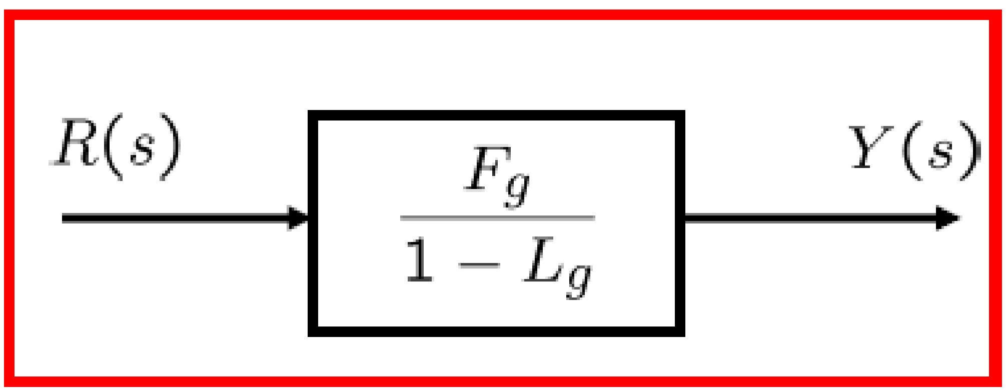
$$\left(\begin{array}{ll} G(s) & \text{: forward path TF} \\ G(s)H(s) & \text{: open-loop TF} \end{array} \right)$$

Feedback loop formula, $TF_{R \rightarrow Y}$



The loop gain is the product of all transfer functions that form the loop

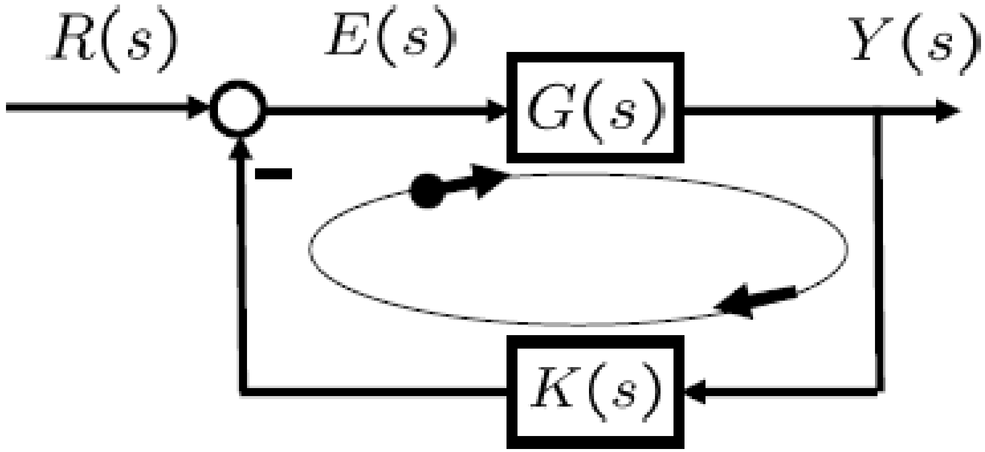
- F_g: Forward gain from R(s) to Y(s) ➡ G(s)
- L_g: Loop gain: ➡ G(s)K(s)(-1)



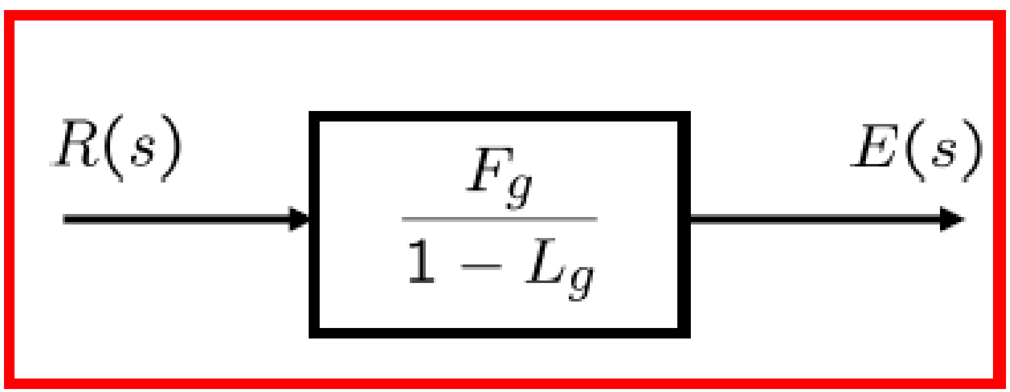
➡

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)K(s)}$$

Feedback loop formula, $TF_{R \rightarrow E}$

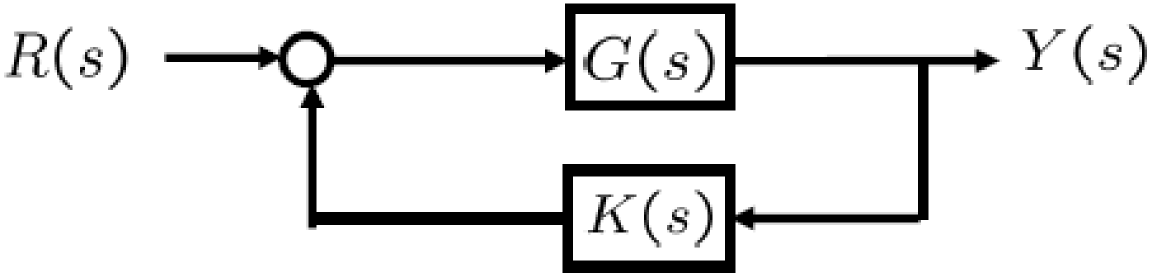


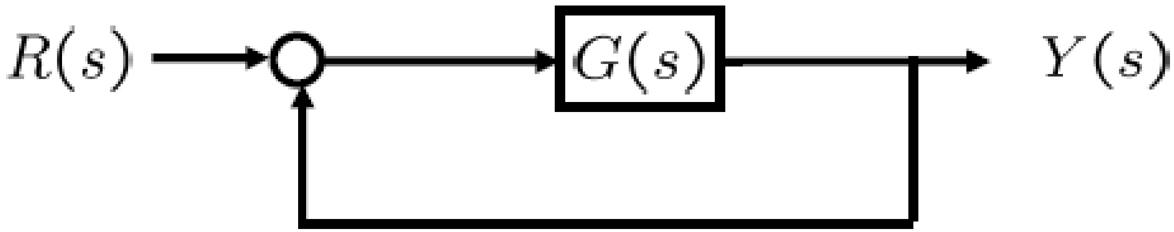
- F_g: Forward gain from R(s) to E(s) ➡ 1
- L_g: Loop gain: ➡ G(s)K(s)(-1)

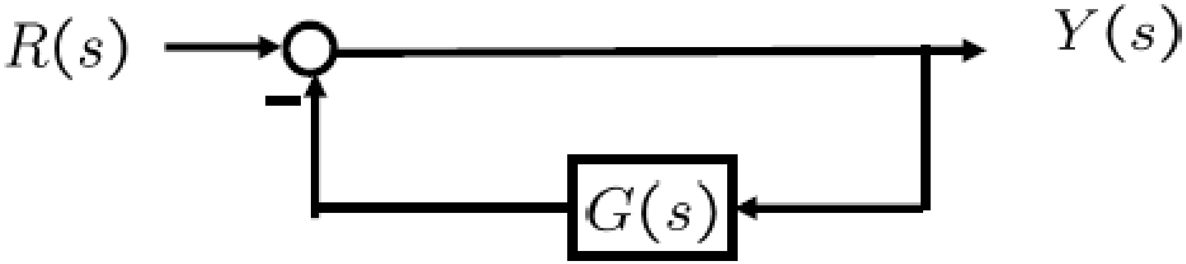


➡ $$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)K(s)}$$

Exercises

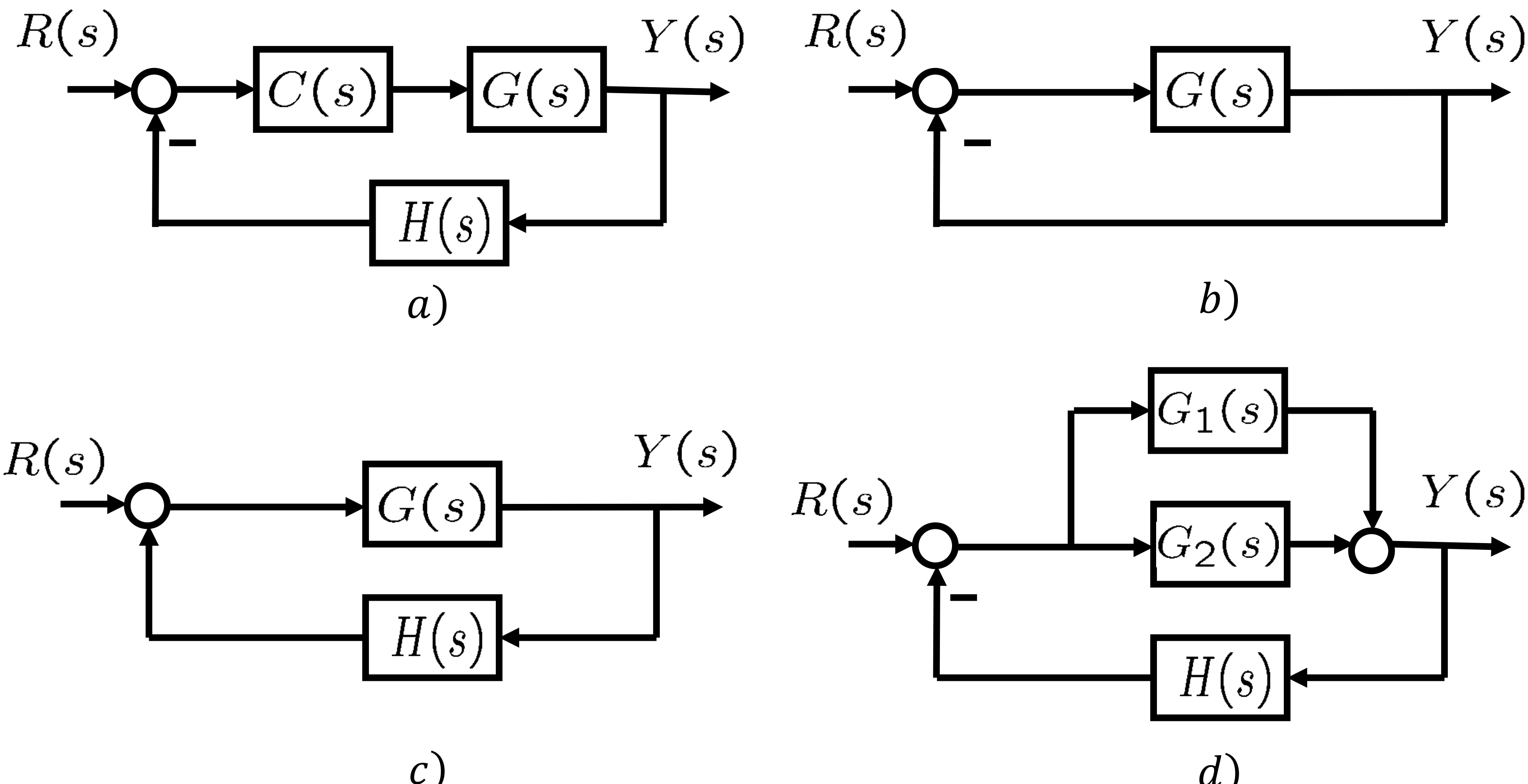






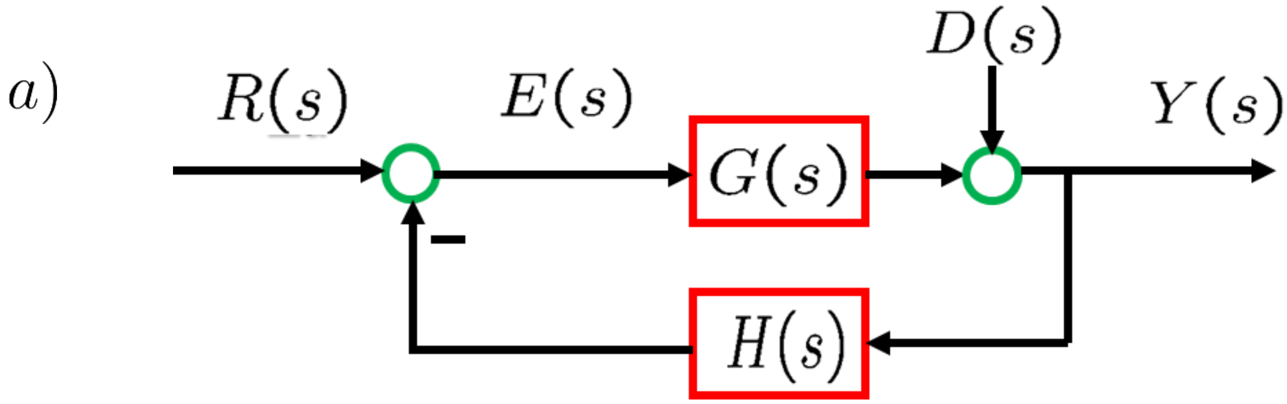
Ex: TF of feedback systems

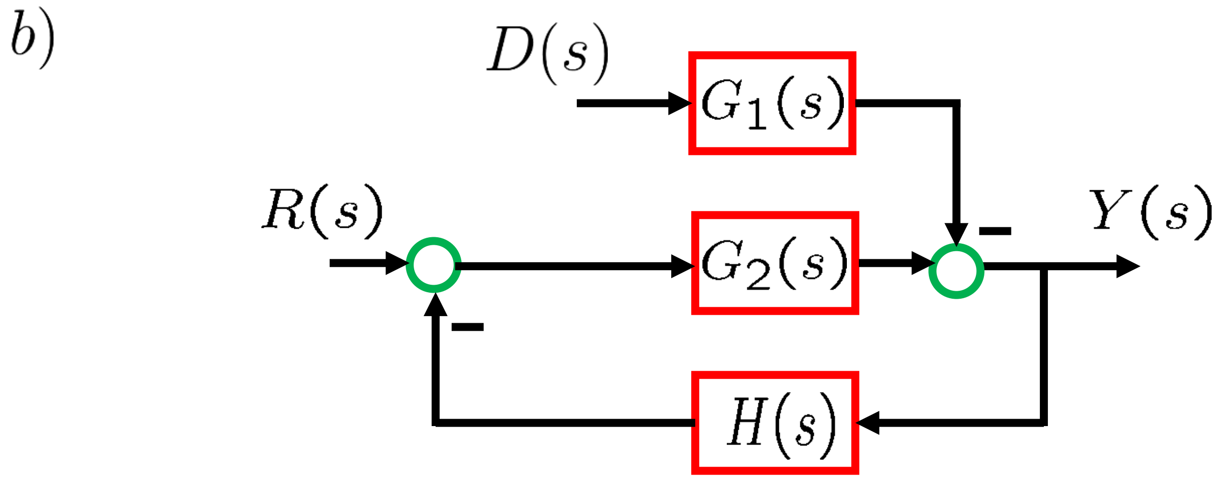
- Compute transfer functions from *R(s)* to *Y(s)*.



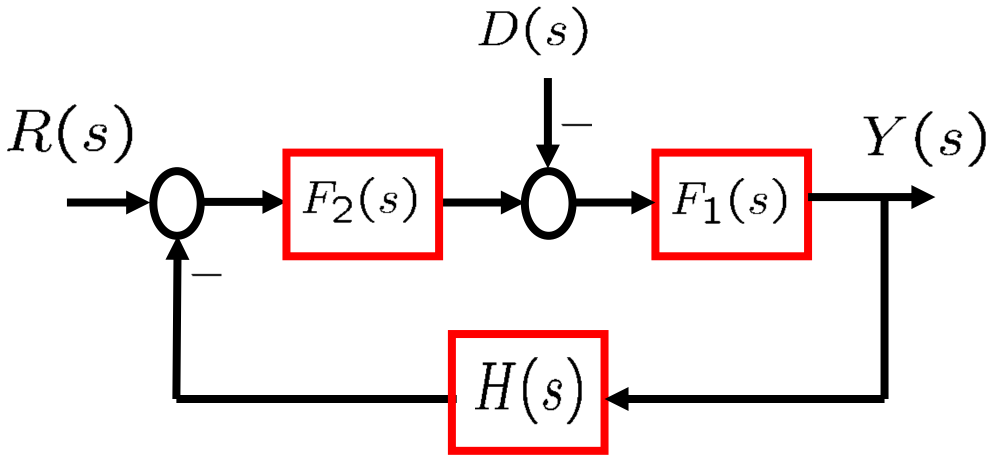
Ex: TF of feedback systems

- Compute transfer function from *D(s)* to *Y(s)*.





$c)$



Summary

- Block Diagrams
  - Multiple blocks, summers
  - Application of negative feedback
  - Overall closed loop transfer function via block diagram reduction
- Next lecture, time response introduction